SISTEMAS DE ECUACIONES Ficha 2

Resuelve los siguientes sistemas de ecuaciones analíticamente:

1.- $\left.\begin{array}{l} y = 3x + 1 \\ \sqrt{x + y + 4} = y - x \end{array}\right\}$

2.- $\left.\begin{array}{l} 2x + y = 6 \\ \sqrt{x} - y = -3 \end{array}\right\}$

3.- $\left.\begin{array}{l} \dfrac{3}{x} - \dfrac{x}{y} = 0 \\ 2x - y = 3 \end{array}\right\}$

4.- $\left.\begin{array}{l} \dfrac{2}{x} + \dfrac{3}{y} = 3 \\ x + y = 4 \end{array}\right\}$

5.- $\left.\begin{array}{l} \dfrac{1}{x + y} = \dfrac{2}{5} \\ \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{5}{2} \end{array}\right\}$

SOLUCIONES

1.-

$$\left.\begin{array}{l} y = 3x+1 \\ \sqrt{x+y+4} = y-x \end{array}\right\} \quad \begin{array}{l} y = 3x+1 \\ \hline \sqrt{x+3x+1+4} = 3x+1-x \end{array}$$

$$\sqrt{4x+5} = 2x+1; \quad 4x+5 = (2x+1)^2$$

$$4x+5 = 4x^2+1+4x; \quad 4 = 4x^2; \quad x^2 = 1$$

$$x = \pm\sqrt{1} \quad \rightarrow \quad \begin{cases} x = -1 \quad \rightarrow \quad \text{no válida} \\ x = 1 \quad \rightarrow \quad y = 4 \end{cases}$$

Hay una solución: $x = 1; \quad y = 4$

2.-

$$\left.\begin{array}{l} 2x+y = 6 \\ \sqrt{x} - y = -3 \end{array}\right\} \quad \left.\begin{array}{l} y = 6-2x \\ \sqrt{x}+3 = y \end{array}\right\} \quad \begin{array}{l} 6-2x = \sqrt{x}+3 \\ 3-2x = \sqrt{x} \end{array}$$

$$(3-2x)^2 = \left(\sqrt{x}\right)^2; \quad 9+4x^2-12x = x; \quad 4x^2-13x+9 = 0$$

$$x = \frac{13 \pm \sqrt{169-144}}{8} = \frac{13 \pm \sqrt{25}}{8} = \frac{13 \pm 5}{8} \quad \rightarrow \quad \begin{cases} x = \dfrac{18}{8} = \dfrac{9}{4} \quad \rightarrow \quad \text{no válida} \\ x = 1 \quad \rightarrow \quad y = 4 \end{cases}$$

$$\left(\text{La solución } x = \frac{9}{4} \text{ no es válida, puesto que } 3 - 2\cdot\frac{9}{4} = -\frac{3}{2} \neq \sqrt{\frac{9}{4}} = \frac{3}{2} \right)$$

La única solución del sistema es $x = 1, \quad y = 4.$

3.-

$$\left.\begin{array}{l} \dfrac{3}{x} - \dfrac{x}{y} = 0 \\ 2x-y = 3 \end{array}\right\} \quad \left.\begin{array}{l} 3y - x^2 = 0 \\ 2x-y = 3 \end{array}\right\} \quad \begin{array}{l} y = \dfrac{x^2}{3} \\ 2x - \dfrac{x^2}{3} = 3; \quad 6x - x^2 = 9 \end{array}$$

$$0 = x^2 - 6x + 9; \quad x = \frac{6 \pm \sqrt{36-36}}{2} = \frac{6}{2} = 3 \quad \rightarrow \quad y = 3$$

Solución: $x = 3; \quad y = 3$

4.-

$$\left.\begin{array}{l}\dfrac{2}{x}+\dfrac{3}{y}=3\\ x+y=4\end{array}\right\}\quad \left.\begin{array}{l}\dfrac{2}{x}+\dfrac{3}{4-x}=3\\ y=4-x\end{array}\right\}\quad \frac{2(4-x)}{x(4-x)}+\frac{3x}{x(4-x)}=\frac{3x(4-x)}{x(4-x)}$$

$$8-2x+3x=12x-3x^2;\quad 3x^2-11x+8=0$$

$$x=\frac{11\pm\sqrt{121-96}}{6}=\frac{11\pm\sqrt{25}}{6}=\frac{11\pm 5}{6}\;\rightarrow\;\left\{\begin{array}{l}x=\dfrac{16}{6}=\dfrac{8}{3}\;\rightarrow\;y=\dfrac{4}{3}\\ x=1\;\rightarrow\;y=3\end{array}\right.$$

$$\text{Hay dos soluciones:}\quad \left.\begin{array}{l}x_1=\dfrac{8}{3}\\ y_1=\dfrac{4}{3}\end{array}\right\}\;\text{y}\;\left.\begin{array}{l}x_2=1\\ y_2=3\end{array}\right\}$$

5.-

$$\left.\begin{array}{l}\dfrac{1}{x+y}=\dfrac{2}{5}\\ \dfrac{1}{x}+\dfrac{1}{y}=\dfrac{5}{2}\end{array}\right\}\quad \left.\begin{array}{l}5=2(x+y)\\ 2y+2x=5xy\end{array}\right\}\quad \begin{array}{l}5=2x+2y\\ 5=5xy\;\rightarrow\;1=xy\;\rightarrow\;y=\dfrac{1}{x}\end{array}$$

$$5=2x+\frac{2}{x};\quad 5x=2x^2+2;\quad 0=2x^2-5x+2$$

$$x=\frac{5\pm\sqrt{25-16}}{4}=\frac{5\pm\sqrt{9}}{4}=\frac{5\pm 3}{4}\;\rightarrow\;\left\{\begin{array}{l}x=2\;\rightarrow\;y=\dfrac{1}{2}\\ x=\dfrac{2}{4}=\dfrac{1}{2}\;\rightarrow\;y=2\end{array}\right.$$

$$\text{Hay dos soluciones:}\quad \left.\begin{array}{l}x_1=2\\ y_1=\dfrac{1}{2}\end{array}\right\}\;\text{y}\;\left.\begin{array}{l}x_2=\dfrac{1}{2}\\ y_2=2\end{array}\right\}$$